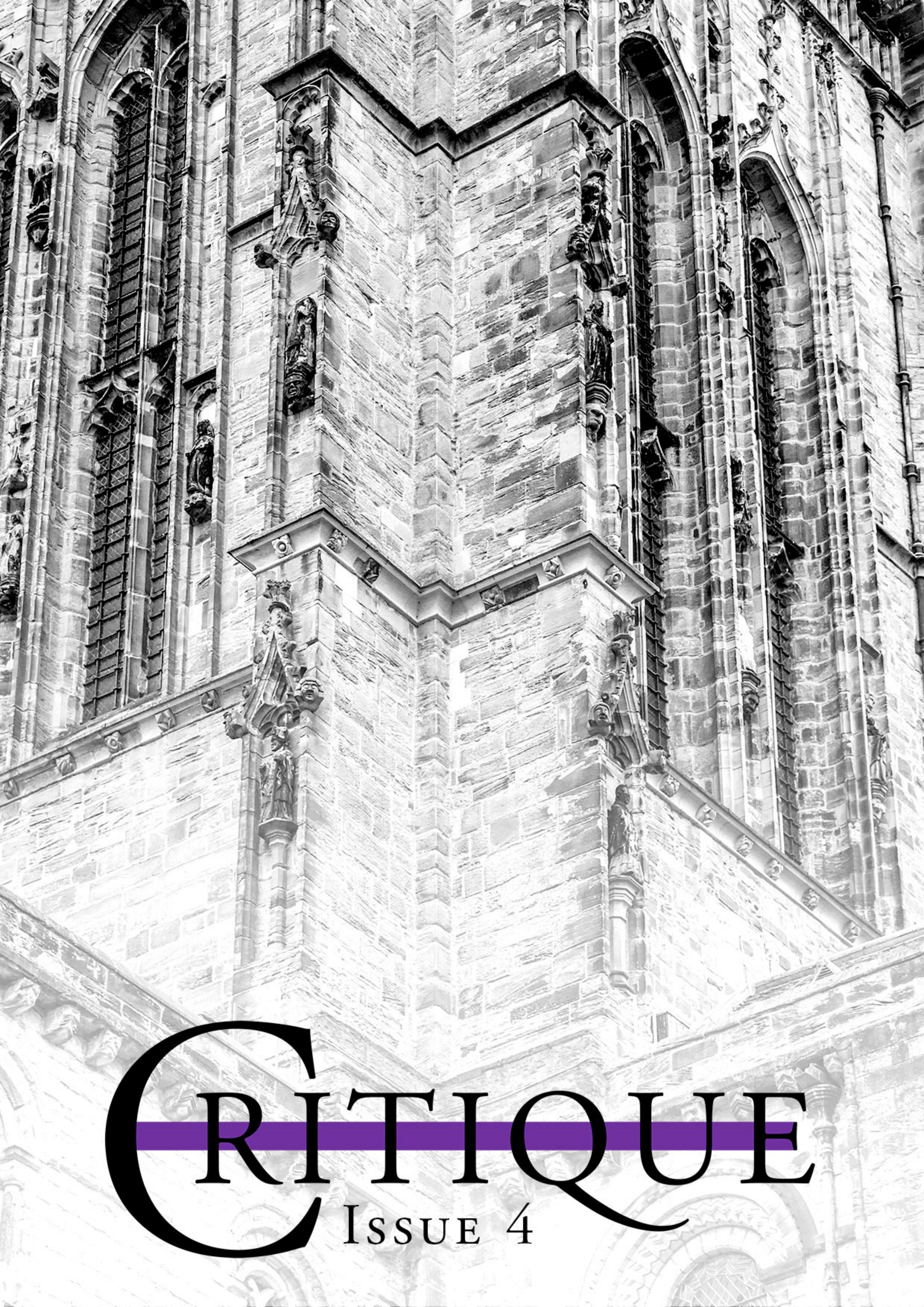

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An Alternative Approach to Mathematical Ontology

Amber Donovan (Durham University)

Introduction

Nominalism and realism are both views in ontology. Thus, what they entail depends on the meta-ontological framework you accept; however, I will define them relative to the truth-making meta-ontological framework in §3. I will ultimately reject this meta-ontological framework and thus, whilst the position I will defend rejects the need for truth-makers for mathematical statements and thus *could* be characterised as nominalist from the truth-making meta-ontological framework, it only does so insofar as it rejects the need for truth-makers for *any* statements.

§1 outlines the Quine-Putnam indispensability argument which takes the meta-ontological principle, Quine's criterion, as P1. Since meta-ontological principles determine meta-ontological frameworks, the use of the Quinean principle indexes the Quine-Putnam indispensability argument to a Quinean meta-ontological framework. As such, §2 demonstrates how Quine's criterion is rejected or amended resulting in two variations of the argument indexed to two contrasting meta-ontological frameworks – deflationary and truth-making. §3 defines nominalism and realism with respect to the truth-making meta-ontology and demonstrates their failure to satisfy both horns of the Benacerraf dilemma. As such, §4 develops a Wittgensteinian account of truth and §5 uses this and the deflationary meta-ontological framework to develop a 'nominalist' position which secures the truth of mathematical statements and satisfies both horns of the Benacerraf dilemma.

§1 The Indispensability Argument

The indispensability argument is commonly taken to motivate a realist position and pose a challenge to the nominalist position. The Quine-Putnam version of the argument is as follows:

> (P1) We ought to have ontological commitment to all and only the entities that are quantified over in our best scientific theory
> (P2) Quantification over mathematical entities is indispensable to our best scientific theories
> (C) We ought to have ontological commitment to mathematical entities
>
> (Baron, 2013:2414)

This formulation of the argument assumes Quine's criterion (P1); a meta-ontological principle which tells us that we should be ontologically committed to those things over which our best scientific theories quantify.

§2 Two Meta-Ontological Frameworks

Philosophers on all sides of the debate have been inclined to either reject or revise Quine's criterion and hence the indispensability argument. I will deal with two principles and opposing instances of this.

§2.1 Deflationary Meta-Ontology

First, I shall make a distinction between two types of numbers: mathematese numbers which can be squared, multiplied, divided, etc. and realist numbers which add to this that numbers are non-spatiotemporal, acausal and eternal. Mathematese numbers are distinct from realist numbers, as whilst we would not say that they are spatially located, we equally need not say, as the realist does, that they are non-spatiotemporal. Collins (1998) makes this explicit in arguing that just as we would not say of the king in chess that it was married, we would also not say that it was a bachelor (Collins, 1998:30). Thus, whilst something cannot be both married and a bachelor, some things are not the sort of thing that can be either. Thus, numbers whilst they are not spatially located, need not be said to be non-spatiotemporal. It suffices to characterise them as mathematese numbers and say that they exist as just that. P2 of the indispensability argument tells us which entities we quantify over indispensably, and P1 dictates that we be committed to *exactly these* entities. However, the numbers over which we quantify indispensably in science need only be mathematese numbers. Thus, the indispensability argument alone does not dictate that we be ontologically committed to anything more than mathematese numbers, despite Quine's criterion being often interpreted as such.

Price (2011) argues that for Quine "there is no space between ontological commitment – belief that there are mathematical entities – and acceptance of quantification over mathematical entities" (Price, 2011:297). As such, he argues that at the point where you accept P2, you not only believe in mathematical entities but are justified in doing so since in quantifying over these entities, you accept their existence (ibid.). Since ontological commitment is equated with quantification over entities, if something stronger is meant by ontological commitment – i.e. the realist sense – then since quantification over entities is meant in the trivial scientific sense, some further argument for why this is evidence for a stronger version of ontological commitment is required (ibid.:298). Thus, further argumentation is required for Quine's criterion to entail the realist conclusion. As such, on this reading we quantify over mathematese numbers, and this is all that can be said about ontological commitment.

This may not seem to answer the *real* metaphysical existence question as mathematese numbers say nothing as to what numbers exist *as*. However, Thomasson (2013) expanding on the ideas of Carnap argues that 'real' existence questions are pseudo questions. Following Carnap, Thomasson uses linguistic frameworks to distinguish between internal and external questions (Thomasson, 2013:31). E.g. we can ask the question 'do numbers exist?' internally (i.e. within the mathematical framework) or externally (i.e. outside of said framework). Internal existence questions have 'easy' answers; the affirmative answer to 'do numbers exist?' follows trivially from the statement 'there are numbers between 5 and 9' or similar – these numbers would be mathematese numbers (ibid.:37).

However, since this is not what the metaphysician intends, they must be asking an external question. Thomasson advocates the Carnapian view that to speak about an entity and hence enquire about its existence we must introduce terms (governed by certain rules) for the entity inside the relevant linguistic framework (ibid.:36-7). E.g. talk about numbers is only possible as they were introduced in the mathematical framework; hence the use of number terms is governed by internally defined rules for their use. If we ask an existence question using 'number' per these rules, we get the 'easy' answer. Since this is not what the 'real' question is meant to be asking, it must be attempting to sever 'number' from this use (ibid.:39). Consider the question "are there fodhsuhd?", this would surely be dismissed as a nonsense since there are no rules to govern the use of this term and hence we have no idea what this means. Thus, if we sever 'number' from its internal usage, the resulting question is no better. Hence, Thomasson concludes that the metaphysical question 'are there numbers?' is a pseudo question (ibid.:34). The only meaningful external questions we can ask are those which decide whether the terms we use and how we use them internally are the best to achieve our purposes, however, again, this is clearly not what the metaphysician intends (ibid.:34).

As such, the indispensability argument under the deflationary meta-ontological framework entails the existence of numbers in a trivial sense. Furthermore, the numbers which exist – as defined by the internal rules for their use – are mathematese numbers, as realist claims that they are 'non-spatiotemporal' arise in an attempt to answer an external pseudo question.

§2.2 Truth-Making Meta-Ontology

Baron (2013) argues that a route available to those who wish to reject the realist conclusion, by rejecting the indispensability argument, is to reject Quine's criterion. Baron contends that such attacks only defeat the indispensability argument if there is no other acceptable criterion available. As such, he proposes to replace it with an Armstrongian criterion of commitment and varies the indispensability argument accordingly (Baron, 2013:2415):

> (P1) We ought to have ontological commitment to all and only the truthmakers required by our best scientific theories.
> (P2) The truth of mathematics is indispensable to our best scientific theories.
> (P3) The truthmakers for mathematical statements are mathematical entities
> (C) Therefore, we ought to have ontological commitment to mathematical entities.
>
> (ibid.:2416)

Baron dubs this the truth-maker indispensability argument. He does not defend the Armstrongian criterion explicitly but contends that the position has many advocates.[1] Although he does contend that Quine's criterion yields some counterintuitive results, particularly when confronted with idealisation in science (ibid.: 2418), which the Armstrongian criterion avoids and thus is stronger in this regard. Thus, the truth-making meta-ontological framework rejects Quine's criterion in favour of an Armstrongian criterion and thus reformulates the indispensability argument.[2]

§3 Realism and Nominalism

Realism and Nominalism are best understood as first-order ontological theses within the second order truth-making meta-ontological framework[3] which differ over the nature of numbers. The realist contends that abstract mathematical entities are the truth-makers for mathematical statements whilst the nominalist contends either (a) there are no truth-makers for mathematical statements or (b) the truth-makers for mathematical statements are non-abstract entities.

However, all three of these positions fail to satisfy both horns of the Benacerraf dilemma. The semantic horn dictates that we have a uniform semantics across natural language and mathematics – viz. whatever semantic theory is accepted for natural language, must also apply to mathematics. Semantic theories, as Benacerraf conceives of them, need to say something about what it is for a statement to be true, and hence,

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[1] Such as Armstrong (2004), Heil (2003), Cameron (2008) and Dyke (2008).

[2] There are naturally objections to such a position, however, I shall not examine them here as I wish to examine the consequences of these positions.

[3] Since, as Collins (1998) notes, it is accepted by both realist and nominalists alike that ontological commitment to mathematical entities entails 'positing' their existence as abstract objects which are acausal and non-spatiotemporal (Collins, 1998:23), these ontological positions cannot be made sense of against a deflationary meta-ontological position.

the semantic horn also dictates that this be uniform across mathematics and natural language. For the epistemological horn to be satisfied we must have account of mathematical truth which fits with a reasonable epistemology (Benacerraf, 1973).

As such, to satisfy the semantic horn of the dilemma, any theory which accepts the truth-making meta-ontology, and hence truth-maker theory, must treat mathematical statements and statements in natural language in the same way. Both nominalist positions fail in this regard. By truth-maker theory, in natural language all true statements have truth-makers, however, nominalism-(a) breaks with truth-maker maximalism[4] and contends that mathematical statements are true and yet do not have truth-makers, thus sacrificing uniform semantics (ibid.:2423). On the other hand, nominalism-(b) contends that mathematical statements have truth-makers but that these are not numbers. Since in natural language, the truth-maker for a true sentence (truth-bearer) is *guided* by the referring terms in the truth-bearer (ibid.:2421) – e.g. the truth-maker for 'the apple is red' should be the apple, or particles arranged apple-wise, in question – if the truth-makers for mathematical statements are not mathematical entities then this principle of guidance is broken. Thus, the truth-makers for mathematical statements work differently and we once again sacrifice uniform semantics (ibid.). As such, both nominalist positions which accept the truth-making meta-ontology fail to satisfy the semantic horn of the Benacerraf dilemma.

The realist position struggles to satisfy the epistemological horn. The realist posits the existence of abstract mathematical entities to serve as truth-makers for mathematical statements. However, mathematical entities as conceived of by the realist are acausal and non-spatiotemporal and as such we cannot interact with them. This presents an epistemological problem as we need some account of how we come to know things about these abstract entities which are independent from us (Field, 1989:68). Whilst this is not an insurmountable problem, the realist is tasked with providing an account of how we know about these things which fits with a reasonable epistemology – this is one of the biggest challenges to the realist position (Øystein, 2017). As such, realist and nominalist positions indexed to the truth-making meta-ontological framework cannot satisfy both horns of the Benacerraf dilemma.

§4 A Wittgensteinian Alternative

That neither the realist nor the nominalist position indexed to the truth-making meta-ontological framework can satisfy both horns of the Benacerraf dilemma suggests that there is something wrong with this meta-ontological stance. The deflationary meta-ontology, whilst providing a deflationary account of existence, does not provide an account of mathematical truth. Mathematese numbers cannot serve as truth-makers as their 'existence' is not robust enough, thus, such an account must be supplemented with an alternative account of truth. As such, I will assume a Wittgensteinian account of language[5] to develop an alternative account of truth which, coupled with the deflationary meta-ontology, can satisfy both horns of the Benacerraf dilemma.

I will first briefly outline a Wittgensteinian view of language. Crudely put, this is the view that language gets its meaning from use. To be meaningful, a word must have a correct use; since words do not always have just one correct use, Wittgenstein introduces the notion of language-games which vary from one context to the next: "when language-games change, there is a change in concepts" (Wittgenstein, OC: §65). Each language-game has its own rules for the use of terms. Thus, to successfully communicate we

[4] Cf. Armstrong, (2004:7)

[5] I will not assess this in detail as I have not done so with truth-maker theory. I merely wish to examine the consequences of each position for the current debate.

must obey the rules of the language-game. These rules need not be explicitly stated as we are trained to follow them, rather than having them explicitly explained to us. As such, language is part of a practice – the practice of obeying rules – and hence rests on contingencies (e.g. that we agree and carry on in the same way).

Like natural language, mathematics is a practice which is contingent upon agreement and regularity, without these things, it could not be a practice at all. Without this, confusion would ensue; this would not result in a different mathematics, but in no mathematics at all (Gerrard, 2006:189). Thus, for mathematics to exist, we must agree on the practice and carry it out in the same way. Once we have this agreement in how to do mathematics, we can agree that certain results of our calculations we call 'true'. Calling things 'true' is yet another practice which is contingent upon agreement, however, it is not synonymous with it – the meaning of 'true' is not "what we agree on". Agreement in practice – be it the practice of calling things 'true' or of mathematics – is a necessary condition for mathematics and for calling things 'true'.

With regards to mathematical statements, to correctly call these 'true' the entities need not exist in the realist's sense. Whilst 4 must exist for 'there exists a square number less than 5' to be true, this does not mean that 4 needs to be an abstract acausal, non-spatiotemporal entity which acts as truth-maker for this sentence. As, since we agree on the practice of mathematics (else it could not be a practice at all) we agree on what it is to practice mathematics correctly and hence on what constitutes a correct mathematical statement. From this we know which mathematical statements can be correctly endorsed by being called true. Thus, the truths of mathematics "are determined by a consensus of action: a consensus of doing the same thing, reacting in the same way. There is a consensus but it is not a consensus of opinion" (Wittgenstein, LFM:183-4).

For Wittgenstein, *all* statements – mathematical or otherwise – are meaningful in virtue of our agreement to adhere to the rules of the relevant language-game. In addition to this, *all* truths in natural language are true in virtue of our agreement on the practice of calling things true, thus, truth-makers are not required for *any* truths not just mathematical ones. As such, our mathematical statements need not succeed in referring to some abstract entity to be true. They are true, like statements in natural language, in virtue of our agreement on what constitutes a correct mathematical statement, and in virtue of our agreement that such statements can be correctly called 'true'.

§5 <u>A 'Nominalist' Position</u>

The nominalist and realist positions per §3 both assume the truth-making meta-ontology and fail one horn of the Benacerraf dilemma. If we reject this meta-ontology (and the accompanying theory of truth) and adopt the deflationary meta-ontology coupled with a Wittgensteinian account of truth then a new position emerges which I argue satisfies *both* horns of the Benacerraf dilemma. The Wittgensteinian account does not require that there be mathematical entities for our mathematical statements to be true. However, this does not mean that numbers therefore do not exist at all, they need not exist in the realist sense, but, even the deflationary reading of the indispensability argument entails the conclusion that numbers exist – as mathematese numbers. Whilst realist numbers come with epistemological problems, mathematese numbers do not. All there is to be known about mathematese numbers is that they can be squared, multiplied, etc. All of this is learnt as we are trained in the practice of mathematics and hence in how to *do* these things with these numbers. There is no epistemological mystery, merely an empirical pedagogical question about how we teach our children to calculate. Thus, this position satisfies the epistemological horn of the Benacerraf dilemma.

Furthermore, Wittgenstein's treatment of the truth and meaningfulness of mathematical statements is the same as statements in natural language; hence this view satisfies the semantic horn of the Benacerraf dilemma as it has a uniform semantics. From the deflationary meta-ontological framework, this position is a trivial realist position. However, since this position contends that there are no truth-makers for mathematical statements, from the truth-making meta-ontological framework it can be categorised as a nominalist position; although, unlike the nominalism-(a) which treats mathematical statements as an exception in this regard, this position rejects the need for truth-makers at all.

Concluding Remarks

I have argued that nominalism and realism are best understood as first-order ontological theses within the second order truth-making meta-ontological framework. However, I have shown that these positions fail to satisfy both horns of the Benacerraf dilemma. Thus, I have argued that we ought to prefer the deflationary meta-ontological framework, as, coupled with a Wittgensteinian view of truth it yields a position which satisfies both horns of the Benacerraf dilemma. Since this position rejects truth-makers for mathematical statements it can be classified as a nominalist position from the truth-making meta-ontological framework – although from the deflationary meta-ontological framework it would be classed as a trivial realist position. Accepting this position to be a nominalist position, I side with the nominalist on the nature of numbers.

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